Biofilter Media Gas Pressure Loss as Related to Media Particle Size and Particle Shape

Lorenzo Pugliese; Tjalle G. Poulsen; and Rune R. Andreasen

Abstract: Pressure loss ($\Delta P$) is a key parameter for estimating biofilter energy consumption. Accurate predictions of $\Delta P$ as a function of air velocity ($V$) are, therefore, essential to assess energy consumption and minimize operation costs. This paper investigates the combined impact of medium particle size and shape on the $V$-$\Delta P$ relationship. The $V$-$\Delta P$ measurements were performed using three commercially available materials with different particle shapes: crushed granite (very angular particles), gravel (particles of intermediate roundness), and lightweight clay aggregate (almost spherical particles). A total of 21 different particle-size fractions, with particle sizes ranging from 2 to 14 mm, were considered for each material. As expected, $\Delta P$ decreased with increasing particle size in agreement with earlier findings. The value of $\Delta P$, however, also showed a tendency to decrease with increasing particle roundness especially for fractions containing smaller particles. A new model concept for estimating $V$-$\Delta P$ across different particle-size fractions and shapes was proposed. This model yielded improved prediction accuracy in comparison with existing prediction approaches. DOI: 10.1061/(ASCE)EE.1943-7870.0000771. © 2013 American Society of Civil Engineers.

CE Database subject headings: Air flow; Granular media; Permeability; Particle size distribution; Moisture; Filtration.

Author keywords: Air flow; Granular media; Permeability; Pressure loss; Particle-size distribution; Particle shape; Moisture; Filtration.

Introduction

Biofiltration is one of the most widely employed technologies for removal of organic pollutants (including malodorous compounds) in air streams originating from industrial and commercial activities (Goldstein 1996; Nicolai and Janni 2000). The filtration process is carried out by the use of microorganisms that are immobilized in a biofilm attached to a porous packing (carrier) material such as straw, wood chips, pebbles, or various artificial materials. Removal of the contaminants then occurs as a consequence of the microbial metabolism where the contaminants are degraded to yield carbon and energy for microbial growth. Biofiltration cost efficiency is generally defined as the quantity of air cleaned to a required level per amount of operation costs. Filter cleaning capacity depends on the quantity of active biomass in the filter, which in turn depends on the active specific surface of the packing material. Operation costs are generally a large part connected with the energy consumption of the filter, which in turn is mainly associated with the pressure loss across the filter. Thus, material physical properties, such as specific surface area and airflow resistance, are key parameters to consider when assessing biofilter performance (Wani et al. 1997; McNevin and Barford 1996; Kim et al. 2000; Elias et al. 2003; Baron et al. 2004). By choosing a proper porous medium, it is possible to increase active microbial biomass (De Oliveira et al. 2009) and contaminant removal capacity (Sakurna et al. 2006) and to reduce the energy consumption associated with the airflow resistance of the biofilter (Malhautier et al. 2005; Gadai-Mawart et al. 2010).

Filter pressure loss ($\Delta P$) is a key parameter for estimating biofilter energy consumption. Accurate predictions of $\Delta P$ are, therefore, necessary when designing biofilters, selecting biofilter packing materials, and estimating economic costs. Darcy (1856) proposed a linear expression relating $\Delta P$ with packing material properties (permeability) based on studies of water flow in filter sand. Several studies have subsequently investigated the relationship between permeability, particle size, and porosity for porous media (Kozey 1927; Fair and Hatch 1933; Carman 1937; Scheidegger 1966; Sorrentino and Anlauf 1999; Sperl and Treckova 2008; Hamamoto et al. 2009).

Forchheimer (1901) observed that in porous media at high fluid flow velocity ($V$), the relationship between $V$ and $\Delta P$ was not linear as predicted by Darcy’s law but instead followed a second-order relationship. Therefore a quadratic $V$ term was added to the Darcy equation to take the effects of inertial forces and turbulence into account. Ergun (1952) proposed a Forchheimer-based expression that essentially links the $\Delta P$ to fluid-filled porosity ($\varepsilon$), a medium particle specific characteristic length ($D_{eq}$) and a set of empirical constants that were supposed to be universal across different porous materials. Ahmed and Sunada (1969) proposed a rearrangement of the Forchheimer equation using the Navier-Stokes equations. The new relationship showed that the empirical constants were related to the properties of the fluid and the porous media by the intrinsic permeability and a proportionality factor (Chin et al. 2009). Ward (1966) derived the same equation by using a dimensional analysis. Macdonald et al. (1979) evaluated the accuracy of the Ergun equation. A large number of experimental data from different porous media were used. The results of Macdonald et al. (1979) showed that the empirical constants in the equation were not independent of porous medium properties, thus, complicating the use of...
the Ergun equation across different types of porous media. Trussell and Chang (1999) analyzed the validity of the Forchheimer and the derived relations on a large set of data (glass beads, anthracite, sand, potter’s beads, marbles, and fragments of crushed dolomite) with respect to impact of medium particle size on $\Delta P$. The results pointed out difficulties in determination of the empirical model parameters, thus, leading to prediction errors. Andreassen and Poulsen (2013) investigated $\Delta P$ in a set of coarse-grained porous materials (with a particle diameter greater than or equal to 2 mm) relevant for biofiltration and found that $\Delta P$ was only weakly dependent on air-filled porosity ($\varepsilon$) for this type of media. These authors, therefore, developed a correlation for predicting $\Delta P$ based on particle-size distribution and particle diameter rather than on material air-filled porosity as traditionally done. The model, developed and tested using a set of media with a wide range of particle sizes, yielded predictions equivalent to 90% reduction in prediction error compared to the Ergun equation. The media tested, however, all belonged to the same material [Leca (Weber AS, Denmark), a granular material used for insulation and biofiltration consisting of porous rounded particles] and there is thus, a need to verify if this model concept is applicable for others types of porous materials, including materials with different particle shapes.

Particle shape influences many physical properties of porous materials such as void ratio, internal friction angle, and air permeability (Witt and Brauns 1983; Shimohara et al. 2000; Rouse et al. 2008). Studies by Connell et al. (1999), Endo et al. (2001), and Pugliese et al. (2012) indicate that particle shape does have an impact on the $\Delta P/L$ relationship, although at present very little is known about how particle shape affects $\Delta P$ in porous media.

The aim of this study is, therefore, to investigate the effects of particle shape on the $\Delta P/L$ relationship across porous materials with different particle shapes and particle-size distributions relevant for biofiltration. The investigation will be based on the model concept for predicting $\Delta P$ in three different commercially available granular materials, which have very different particle shapes: crushed granite (very angular particles), gravel (particles of intermediate roundness), and Leca (almost spherical particles). A total of 21 different particle-size fractions, with particle sizes ranging from 2 to 14 mm were considered for each of the three materials.

Theory

At low $V$, the $\Delta P$ of a fluid flowing through a porous medium can be described by Darcy’s law (Darcy 1856) which, however, is only valid for Reynolds numbers (Re) (Andreassen et al. 2012) below approximately 1, when flow conditions are laminar and the inertial forces in the flow field are negligible. At $\Re > 1$ (higher flow $V$), inertial forces are important, the $\Delta P$ relationship becomes nonlinear and Darcy’s law no longer applies. Several nonlinear equations for relating $V$ and $\Delta P$ in this region of $\Re$ have been presented (Green and Duwez 1951; Cornell and Kazi 1953; Geertsma 1974; Antohe et al. 1997; Lage et al. 1997; Trussell and Chang 1999) but the most widely used is the second-order Forchheimer relationship (Forchheimer 1901). This relationship was originally developed for $1 < \Re < 100$, but it can also be used to approximate the $\Delta P$ relationship above $\Re = 100$ (Trussell and Chang 1999; Andreassen and Poulsen 2013).

Among the expressions describing fluid flow through packed beds following the Forchheimer relationship, the Ergun equation (Ergun 1952) is perhaps the most widely used. This equation uses an equivalent particle diameter ($D_{eq}$) and three empirical constants for which Ergun suggested universal values. Macdonald et al. (1979) later tested the Ergun equation against a large set of flow-pressure data from several porous media and found that the empirical constants depended on the physical properties of the material. Macdonald et al. (1991) proposed an expression for estimating $D_{eq}$ using the first- and the second-order moments of the particle-size distribution. A more thorough presentation of the preceding theory and the equations involved can be found in Andreassen et al. (2012), Andreassen and Poulsen (2013), and Andreassen et al. (2013).

A simpler expression to evaluate $D_{eq}$ was proposed by Andreassen and Poulsen (2013). Based on measurements for a large set of porous media with uniform particle-size distributions originating from the same material (Leca, with diameters ranging from 2 to 18 mm) $D_{eq}$ for materials with uniform particle-size distributions, was estimated as a harmonic mean of the mean particle diameter ($D_m$) and the minimum particle diameter ($D_{min}$) for each medium as

$$D_{eq} = \frac{2}{\frac{1}{D_m^2} + \frac{1}{D_{min}^2}}$$  \hspace{1cm} (1)

Andreassen and Poulsen (2013) further observed that $\Delta P$ in these media was almost independent of air-filled porosity and therefore suggested that $\Delta P$ can be predicted as

$$\frac{\Delta P}{L} = A\left(\frac{2}{V_{s}^{2}} + \frac{1}{\mu_{s}^{2}}\right)^{-2} + B\left(\frac{2}{V_{s}^{2}} + \frac{1}{\mu_{s}^{2}}\right)^{-1} \rho V^{2}$$  \hspace{1cm} (2)

where $\Delta P$ is the pressure drop across the medium (Pa); $L$ is the distance over which the pressure drop takes place (m); $\mu$ is the air viscosity (Pa s); $\rho$ is the air density (kg/m$^3$); $V$ is the superficial air velocity (m/s); and $A$ and $B$ are empirical constants. Based on this study, the authors concluded that a likely dependency of $\Delta P/L$ on particle shape was expected. The authors also suggested that instead of defining $D_{eq}$ based on the smallest and largest particle diameter, improved predictions might be achieved by using alternative values.

Particle shape is often characterized by particle roundness, which is the ratio between the diameters of the largest inscribed and the smallest circumscribing spheres (Santamarina and Cho 2004). Earlier studies (Krumein 1941; Meloy 1977; Barret 1988; Bowman et al. 2001) have concluded that the projection sphericity (equivalent to roundness) (Cox 1927; Pendleton 1927; Tickell 1913; Wadell 1935) represents the best way to analyze effects of particle shape on transport in porous media. Two very widely applied approaches are those of Pendleton (1927) and Wadell (1935). Pendleton (1927) defined the roundness as

$$\varphi_p = \frac{\omega_p}{\omega_p^{\omega_p}}$$  \hspace{1cm} (3)

where $\varphi_p$ is the total degree of roundness (dimensionless) based on Pendleton (1927) which cannot be greater than 1; $\omega$ is the cross-sectional or projection area of the grain (m$^2$); and $\omega_p$ is the area of the circle having the largest diameter of the grain (m$^2$). The orientation of the particles was not definite.

Wadell (1935) defined the roundness as

$$\varphi_n = \frac{N}{\sum_{N}(1/N)}$$  \hspace{1cm} (4)

where $\varphi_n$ is the total degree of roundness based on Wadell (1935) (dimensionless); $N$ is the number of corners in the given plane of the particle; $R$ is the radius of the largest inscribed circle (m); and $r_n$ is the radius of the inscribed circle of the $N$th corner of the particle in the plane (m).

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Materials and Methods

Measurements of $\Delta P$ were conducted using three commercially available materials: crushed granite, gravel, and Leca (light expanded clay aggregates). An irregular and very angular particle shape characterizes the crushed granite; gravel consists of somewhat rounded rock fragments and Leca consists of rounded particles. Granite and gravel do not have any internal porosity (inside the particles) while Leca consists of highly porous particles; although this internal porosity is inaccessible by air as it consists of closed vesicles very much like soap foam. Fig. 1 shows the three materials.

All three materials were initially sieved into six particle-size fractions with six particle-size distributions. Each of these fractions was characterized by a particle-size range ($R$) of 2 mm in the range between 2 and 14 mm. Particle diameters ($D$) were $2 \leq D < 4$, $4 \leq D < 6$, $6 \leq D < 8$, $8 \leq D < 10$, $10 \leq D < 12$, and $12 \leq D < 14$ mm corresponding to mean particle diameter ($D_m$) of 3.5, 7, 9, 11, and 13 mm, respectively. Additional fractions with $R = 4$ mm ($D_m = 4, 6, 8, 10, 12$ mm); $R = 6$ mm ($D_m = 5, 7, 9, 11$ mm); $R = 8$ mm ($D_m = 6, 8, 10$); $R = 10$ mm ($D_m = 7, 9$); and $R = 12$ mm ($D_m = 8$), with uniform particle distributions, were produced by combining appropriate quantities of the six $R = 2$ mm fractions. Uniform particle-size distributions were chosen to ensure well-defined particle-size distributions across all particle-size fractions used. A total of 63 particle-size fractions were produced (21 for each material).

For granite, $\rho_p$ for each particle size fraction was measured by packing the material into a known volume followed by weighing. The external porosity ($\varepsilon_{ex}$), which for granite equals $\varepsilon_{ex}$, was calculated from $\rho_p$ using a solid density of 2.75 g/cm$^3$ (Hausarath et al. 2009; Omorsany et al. 2012). Corresponding values of $\rho_p$ and $\varepsilon_{ex}(\varepsilon_{ex})$ for gravel and Leca were obtained from Sharma and Poulsen (2010) and Andreassen and Poulsen (2013), respectively. For determination of roundness, $\varphi_p$ and $\varphi_m$, 30 particles were randomly selected from each of the 3 materials (5 particles for each of the $R = 2$ mm fractions). For all granite and gravel particles, projections of particle shape onto a flat surface were carried out in three perpendicular planes. For Leca, projections were only carried out in one plane as particle shape was observed to be very similar in all planes. Values of $\omega$ and $\varphi_p$, and consequently $\varphi_p$ [Eq. (3)] were subsequently determined from the projections. Values of $R$ and $\varepsilon_{ex}$ and consequently $\varphi_p$ [Eq. (4)] were determined by selecting the two sharpest corners of each projection ($N = 2$) and analyzed using a circle scale as proposed by Wadell (1935). Average values of $\varphi_p$ and $\varphi_m$ were then calculated for each material across the 6 $R = 2$ mm particle-size fractions (five particles for each). All measurements were carried out in duplicate. An overview of the media properties is given in Table 1.

Each experiment was performed by packing each of the 21 particle-size fractions for each of the 3 materials into a clear acrylic column of 100 cm in length and 14 cm inner diameter. This column size was chosen in order to avoid effects of preferential flow along the column walls, which may occur if column diameter is too small, compared to the average particle diameter (Pugliese et al. 2012). Great care was taken to achieve a uniform packing (especially along the length of the column), to reduce variations in $\rho_p$. Measurements of $\Delta P$ were carried out for each of the 63 particle size fractions following the approach of Pugliese et al. (2012). Columns were fitted with a polyethylene lid and sealed with a rubber O-ring at the bottom. A stainless steel mesh with 2-mm openings and 1-mm thickness was installed to maintain a distance of 10 mm between the lid and the porous medium. The top of the column was kept open to the atmosphere while the bottom was connected to a supply of compressed atmospheric air via a valve and a precision ball flowmeter (Model P450; Porter Instrument Div., Hatfield, PA). Soft Teflon tubing with an inner diameter of 4 mm was used to connect system components. Corresponding values of $V$ and $\Delta P$ across the columns were measured for $V = 0.005, 0.010, 0.016, 0.021, 0.032, 0.043, 0.054, and 0.065$ m$^3$/s, equal to $Q = 5, 10, 15, 20, 30, 40, 50$, and $60$ l/min, respectively. The relatively wide $Q$-range was chosen to get more reliable determination of the $V-\Delta P$ relationships for the different media. An Alnor AXD 560 digital manometer (Alnor, Ontario, Canada), connected to the bottom and the top of the column, was used to measure $\Delta P$. Measured $\Delta P$ values were corrected for the pressure drop across the empty column with the metal mesh in place. All experiments were carried out in duplicate. A schematic of the experimental setup is shown in Fig. 2.

Results and Discussion

Measured $V-\Delta P/L$ relationships for selected particle-size fractions for all three materials are shown in Fig. 3.

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Fig. 1. Porous materials used in the tracer experiments: (a) granite; (b) gravel; (c) Leca
Table 1. Physical Properties of the Three Porous Materials (Granite, Gravel, and Leca) and 63 Particle-Size Fractions

<table>
<thead>
<tr>
<th>Size range (mm)</th>
<th>Granite</th>
<th>Gravel</th>
<th>Leca</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_a$ (g/cm$^3)$</td>
<td>$\phi_p$</td>
<td>$\phi_w$</td>
</tr>
<tr>
<td>2-4</td>
<td>2.75</td>
<td>0.64</td>
<td>0.09</td>
</tr>
<tr>
<td>2-6</td>
<td>1.51</td>
<td>0.45</td>
<td>0.42</td>
</tr>
<tr>
<td>2-8</td>
<td>1.55</td>
<td>0.44</td>
<td>0.42</td>
</tr>
<tr>
<td>2-12</td>
<td>1.52</td>
<td>0.45</td>
<td>0.42</td>
</tr>
<tr>
<td>2-14</td>
<td>1.48</td>
<td>0.47</td>
<td>0.42</td>
</tr>
<tr>
<td>4-8</td>
<td>1.55</td>
<td>0.44</td>
<td>0.42</td>
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<td>4-10</td>
<td>1.50</td>
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<td>0.42</td>
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<tr>
<td>4-12</td>
<td>1.53</td>
<td>0.44</td>
<td>0.42</td>
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<tr>
<td>6-8</td>
<td>1.49</td>
<td>0.46</td>
<td>0.42</td>
</tr>
<tr>
<td>6-12</td>
<td>1.46</td>
<td>0.47</td>
<td>0.42</td>
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<tr>
<td>6-14</td>
<td>1.55</td>
<td>0.44</td>
<td>0.42</td>
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<tr>
<td>8-10</td>
<td>1.55</td>
<td>0.45</td>
<td>0.42</td>
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<tr>
<td>8-14</td>
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<tr>
<td>6-14</td>
<td>1.54</td>
<td>0.44</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Fig. 2. Experimental setup for measuring pressure loss ($\Delta P$) in porous filter media

The $\Delta P/L$ relationships follow a second-order polynomial in all cases, thus, being consistent with Eq. (2). This was the case for all materials and particle-size fractions investigated. Values of $\Delta P/L$ range up to 60, covering therefore both the Darcy and Forchheimer flow domains. $\Delta P/L$ generally decreased with increasing average particle size. The reason is that smaller particles means smaller pores and, thus, increasing resistance to flow. The smallest pores of the medium considered here characterize the 2-4 mm fractions (average particle size equal to 3 mm) and, therefore, this medium has the greatest $\Delta P$. In contrast, the lowest values of $\Delta P/L$ were obtained for the 12-14 mm particle size fraction that has the largest average particle diameter (13 mm) and, therefore, also the largest pores.

For identical particle-size fractions, the crushed granite generally yielded the highest $\Delta P/L$ while Leca yielded the lowest values of $\Delta P/L$ despite the fact that granite has a higher external (active) air-filled porosity compared to Leca (Table 1). In fact, for the three materials considered in this study, $\Delta P/L$ was inversely correlated with active air-filled porosity. This supports the findings of Andreassen and Poulsen (2013) who observed that $\Delta P/L$ was independent of active air-filled porosity across a wide range of Leca particle-size fractions. These authors, therefore, suggested to base models for predicting $\Delta P/L$ in coarse-grained (2-18 mm) granular materials on particle diameter rather than air-filled porosity. This is likely because active porosity in fine-grained materials (less than 2 mm) for which Eq. (2) was originally developed, has a very different structure due to for instance cracks and formation of aggregates compared to coarse-grained materials such as those investigated here. Gravel exhibited intermediate $\Delta P$ compared to Leca.
and crushed granite. The granite and gravel $\Delta P / L$ were 1.24 and 1.19 times larger than those for the Leca on average across all particle-size fractions, respectively. In general, the relative differences in $\Delta P / L$ between the three materials were most prominent for the fractions containing smaller particles.

Values of particle roundness for the three materials as calculated by Pedersen (1927) [Eq. (3)] and Wadell (1935) [Eq. (4)] were lowest for granite and highest for Leca. As the principal difference between the three materials considered here is their particle shape (as characterized by roundness), the deviations in $e_\text{ex}$ and $p_\theta$, and $\Delta P / L$ between the three materials likely result from the differences in roundness. Flow in materials consisting of rounded particles (Leca) should be expected to be more laminar and less subject to inertial forces than flow in materials consisting of more angular (less spherical) particles such as crushed granite. Pagliese et al. (2012) observed that, in two materials with identical particle-size distributions but different particle shapes (pebbles with high roundness and crushed slate with low roundness), dispersivity decreased with decreasing particle roundness. This behavior was therefore attributed to the difference in particle shape. The same authors further observed an inverse linear proportionality between dispersivity and $\Delta P / L$ under otherwise identical conditions. Thus $\Delta P / L$ was observed to increase with decreasing particle roundness supporting that the differences in $\Delta P / L$ observed between the three materials used here mainly stem from differences in particle shape. In general the results discussed earlier showed that among the three factors (particle size, particle shape, and porosity) particle size is the most important for controlling $\Delta P / L$ (a well-known fact) followed by particle shape, while porosity has little or no impact on $\Delta P / L$ in coarse-granular materials.

The model [Eq. (2)], suggested by Andreassen and Poulsen (2013) for predicting the $V-\Delta P / L$ relationships in materials with uniform particle-size distributions (developed based on data for 56 Leca particle-size fractions), was tested against the $V-\Delta P / L$ values measured in this study for all three materials (1,008 data points).

The test was performed by fitting Eq. (3) to the measured data using the model constants $A$ and $B$ as fitting parameters. Optimal values of $A$ and $B$ for each of the three materials were identified by minimizing the sum of the relative squared errors (RSE) between measured and fitted $\Delta P / L$ values, calculated as

$$RSE = \sum_{i=1}^{N} \left( \frac{\Delta P / L_{\text{measured}}(i) - \Delta P / L_{\text{predicted}}(i)}{\Delta P / L_{\text{measured}}(i)} \right)^2$$

where $\Delta P / L_{\text{measured}}(i)$ and $\Delta P / L_{\text{predicted}}(i)$ is the observed and predicted (by the model) pressure gradients, respectively; while $N$ is the total number of measurements.

The optimal values of $A$ and $B$ for the three materials are given in Table 2 and the fitted versus measured values of $\Delta P / L$ by Eq. (3) with the $A$ and $B$ values from Table 2 are shown in Fig. 4 for the three materials. Values of $A$ and $B$ for Eq. (3) are relatively similar across the three materials with averages of 443 ($\pm 9\%$) and 56 ($\pm 15\%$), respectively, with numbers in parentheses indicating one standard deviation. In general the model is able to fit the data relatively well with relative RSE values ranging from 0.17 to 0.21 indicating that the average squared error in fitting individual values of $\Delta P / L$ is 0.19 on average.

Fig. 4 shows that it is possible to get a relatively close fit of Eq. (3) to the measured $\Delta P / L$ values especially considering that the $\Delta P / L$ varies over three orders of magnitude. The plots, however, also show that Eq. (2) has a tendency to underpredict small [$-0.5 < \log(\Delta P / L) < 0.5$] and large [$2.0 < \log(\Delta P / L) < 2.5$] $\Delta P / L$ values. This behavior is actually not seen only for the three data sets as a whole but also for the individual particle-size fractions for each material.

This is illustrated in Fig. 5 that shows the relative deviation between measured and fitted $\Delta P / L$ values (calculated using Eq. (2)) as a function of the measured $\Delta P / L$ for selected

<table>
<thead>
<tr>
<th>Approach</th>
<th>Equation</th>
<th>$A_{\text{granite}}$</th>
<th>$A_{\text{gravel}}$</th>
<th>$A_{\text{Leca}}$</th>
<th>$B_{\text{granite}}$</th>
<th>$B_{\text{gravel}}$</th>
<th>$B_{\text{Leca}}$</th>
<th>$\alpha_{\text{granite}}$</th>
<th>$\alpha_{\text{gravel}}$</th>
<th>$\alpha_{\text{Leca}}$</th>
<th>$\text{RSE}_{\text{granite}}$</th>
<th>$\text{RSE}_{\text{gravel}}$</th>
<th>$\text{RSE}_{\text{Leca}}$</th>
<th>$\text{RSE}_{\text{total}}$</th>
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<tr>
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<td>6</td>
<td>Eq. (7) $F$</td>
<td>562</td>
<td>51</td>
<td>68</td>
<td>15</td>
<td>0.17</td>
<td>0.20</td>
<td>0.18</td>
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**Fig. 4.** Fitted [using Eq. (2)] versus measured values of Log $(\Delta P / L)$ for (a) granite; (b) gravel; (c) Leca.
particle-size fractions for each of the three materials. In all six cases shown in Fig. 5, the relative deviation between measured and fitted \( \Delta P/L \) values as a function of the measured \( \Delta P/L \) shows an upward concave relationship. This means that on average the model tends to over predict \( \Delta P/L \) at low and high values of \( \Delta P/L \) and under predict at intermediate \( \Delta P/L \). This tendency was also observed for all other particle-size fractions across all three materials but tended to be most prominent for particle-size fractions containing mainly small or large particles. The reason is that the model (Eq. (2)) is not able to fully capture the nonlinear relationship between \( \Delta P/L \) and \( V \) for the materials. As the curvature of the modeled \( V-\Delta P/L \) relationship by Eq. (2) is not only controlled by the empirical constants, \( A \) and \( B \), but also by the value of \( D_{eq} \), it is likely that a more optimal choice of expression for \( D_{eq} \) may improve model results. Andreassen and Poulsen (2013) proposed that \( D_{eq} \) might be predicted as the harmonic mean of the smallest and the mean particle diameter present in each particle size fraction [as given by Eq. (1)], thus the smallest and the mean particle sizes are weighted equally in the prediction of \( D_{eq} \). Earlier studies, however, have suggested that the characteristics of granular materials might be predicted using the particle diameters corresponding to the 10 and 85% fractiles of the particle-size distribution, \( D_{10} \) and \( D_{85} \), respectively. These particle diameters will also be applicable to nonuniform particle-size distributions while this is not possible when using Eq. (1) as \( D_{eq} \) is often not known for nonuniform particle-size distributions. Thus, the choice of \( D_{min} \) and \( D_{max} \) (corresponding to \( D_{10} \) and \( D_{85} \)) for predicting \( D_{eq} \) is likely not the optimal choice. It is also well known that for a given medium consisting of a mixture of particles of different diameters, the smaller particles have a larger impact on \( \Delta P/L \) than large particles. Therefore, weighting the two diameters equally in the \( D_{eq} \) prediction as done in Eq. (1) may not be the optimal approach.

We, therefore, propose that the following expression for predicting \( D_{eq} \) across different materials:

\[
D_{eq} = \frac{1}{\left( \frac{a}{D_{10}} + \frac{1-a}{D_{85}} \right)}
\]  

where \( a = a \) weighting factor \( 0 < a < 1 \); and \( D_{10} \) and \( D_{85} \) (m) = the particle diameters at which 10 and 60% of the particle mass consists of particles with a smaller diameter, respectively. Replacing Eq. (1) with Eq. (6) in Eq. (7) yields

\[
\frac{\Delta P}{L} = A \left( \frac{1}{D_{eq}} \right)^{2} - B \left( \frac{1}{D_{eq}} \right)^{3} \rho v^2
\]

Eq. (7) was, therefore, fitted to the measured \( V-\Delta P/L \) data for all three materials using A, B, and a as fitting parameters. Six different fitting approaches were tested: (1) values of A, B, and a were fit individually for each of the three materials (a total of nine fitting parameters); (2) A and B were fit individually for each material while one common value of a across all three materials was used (seven fitting parameters); (3) A was fit individually for each material while common values of B and a were used (five fitting parameters); (4) B was fit individually for each material while common values of A and a were used (five fitting parameters); (5) a was fit individually for each material while common values of A and B were used (five fitting parameters); and (6) common values of A, B, and a were used (three fitting parameters). Values of A and B for Eq. (2) and A, B, and a for all six fitting approaches using Eq. (7) are shown in Table 2. Values of the RSE [Eq. (5)] were calculated for each approach both considering each of the three materials individually and all three materials together. These data are also shown in Table 2. Fitted versus measured values of \( \Delta P/L \) using approach (1) for all three materials and all gas velocities considered are shown in Fig. 6(a) for granite, Fig. 6(b) for gravel, and Fig. 6(c) for Leca.

The use of Eq. (7) in Approach 1 reduces the RSE by 25% compared to using Eq. (2). The main difference between Eq. (7) in Approach 1 and Eq. (2) is that individual values of \( D_{eq} \) for each material are used in the former approach. Thus, although the reduction in RSE is relatively modest, this indicates that improved predictions of \( \Delta P/L \) may be achieved by taking into account the effects of particle shape on the value of \( D_{eq} \). Values of A are generally 20–30% higher when using Eq. (7) compared to Eq. (2) while values of B are approximately the same. When using Eq. (7) in Approach 2, there is a clear relationship between A and particle roundness such that A decreases with increasing roundness. For Approaches 1 and 3–6, this tendency is less clear although the lowest values of A are still observed for the highest degree of roundness. Relationships between B, a, and roundness are less clear, although for both parameters the lowest values are observed for the highest degree of roundness indicating that all three parameters A, B, and a are related to particle roundness. Particle roundness may not be the only controlling factor or perhaps not optimal for characterizing particle shape as particles of different shapes may exhibit similar roundness. Also particle characteristics such as surface roughness affect \( \Delta P/L \), A, B, and a. Measurement of particle surface roughness on nonspherical particles remains a challenge due to the difficulty in distinguishing particle angularity and particle surface roughness as these two characteristics have similar impact on \( \Delta P \).

The data in Table 2, further, show that Eq. (7) can achieve the same (or slightly better) prediction accuracy compared to Eq. (2) using only half the number of empirical fitting parameters [compare Eq. (2) with Eq. (7) Approach 6 in Table 2]. Thus, Eq. (7) offers a much simpler approach for predicting \( \Delta P/L \) in granular materials as it only requires one value of A, B, and a regardless of material type, particle shape, and size. The results indicate that A ≈ 562, B ≈ 51, and a ≈ 0.7 are suitable as long as the particle-size distributions of the individual particle-size fractions are uniform. It is very likely that at least some of the empirical parameters A,
Fig. 6. Measured and predicted [Eq. (7) with $A$, $B$, and $a$ fitted individually for each material] values of $\Delta P/L$, for (a) granite; (b) gravel; (c) Leca

$B$, and $a$ depend on the shape of the particle-size distributions considered, however, more $\Delta P/L$ measurements in media with different particle-size distribution shapes, originating from the same material are needed to verify this.

Conclusions

$\Delta P$ as a function of air $V$ was investigated in three different commercially available granular materials, which have very different particle shapes: crushed granite (very angular particles, low roundness); gravel (particles of intermediate roundness); and Leca (almost spherical particles, high roundness). A total of 21 different particle-size fractions were considered, with particle sizes ranging from 2 to 14 mm for each of the three materials.

Overall 1,008 $V\Delta P/L$ measurements have been carried out. $V\Delta P/L$ followed a second-order polynomial in agreement with earlier studies. Results showed that $\Delta P/L$ decreased with increasing particle size and that it was inversely correlated with active air-filled porosity. The latter is in contrast to earlier findings for fine-grained materials such as soils and is likely because the pore structure for fine-grained materials is different from coarse-granular materials used here. Values of $\Delta P/L$ generally decreased with particle roundness. For granite and gravel, $\Delta P/L$ was 1.24 and 1.19 times larger than for Leca on average. Differences in $\Delta P/L$ between the three materials were most prominent for the fractions containing smaller particles. This supports the hypothesis that at least part of the differences in $\Delta P$ observed between the three materials used here stems from differences in particle shape (roundness).

An existing model for predicting $\Delta P/L$ as a function of $V$, developed for Leca, was tested against the $V\Delta P/L$ values measured in this study, for all three materials. The model performed relatively well although it had a tendency to underpredict small and large while overpredicting intermediate $\Delta P/L$ values. This was the case for the three data sets as a whole but also for the individual particle-size fractions for each material. Thus, an improved model for predicting the $V\Delta P/L$ relationship was proposed.

This model was able to yield the same accuracy of $V\Delta P/L$ predictions as the existing model using only half the number of model-fitting parameters (three instead of six) and is, thus, simpler to apply. Results from the modeling further indicated that the material-specific empirical parameters were related to particle shape. Further measurements on other materials with a wider range of particle-size distribution and particle shapes, however, are required to better understand the relationship between the $V\Delta P/L$ relationship for a given material and its particle size distribution, particle shape, and other characteristics such as particle surface roughness.

References


